1) Solution:

a) Derive the transfer function

**Method 1 (Laplace Transform approach)**

We have

\[ \dot{x}_1(t) = -3x_1(t) + x_2(t) + u(t), \]
\[ \dot{x}_2(t) = -2x_2(t) + 2u(t), \]
\[ y(t) = x_1(t) + x_2(t). \]

Taking the Laplace transforms of the above equations we get

\[ [s + 3]X_1(s) = X_2(s) + U(s), \]
\[ [s + 2]X_2(s) = 2U(s) \]
\[ \Rightarrow X_2(s) = \frac{2}{s + 2}U(s). \]

Therefore, we have

\[ X_1(s) = \frac{2}{(s + 3)(s + 2)}U(s) + \frac{1}{s + 3}U(s), \]
\[ \Rightarrow X_1(s) = \frac{(s + 4)}{(s + 3)(s + 2)}U(s). \]

Hence

\[ Y(s) = X_1(s) + X_2(s) = \frac{s + 4}{(s + 3)(s + 2)}U(s) + \frac{2}{s + 2} \]
\[ \Rightarrow \frac{Y(s)}{U(s)} = \frac{(3s + 10)}{(s + 3)(s + 2)}. \]

**Method 2 (State Space approach)**

The transfer function of the system can also be computed by using the formula given by

\[ \frac{Y(s)}{U(s)} = C(sI - A)^{-1}B + D. \]

In this problem, we have
\[ A = \begin{bmatrix} -3 & 1 \\ 0 & -2 \end{bmatrix}, \]
\[ B = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \]
\[ C = [1 \ 1], \]
\[ D = 0. \]

Therefore, we have
\[ \frac{Y(s)}{U(s)} = \left[ 1 \ 1 \right] \left( \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} -3 & 1 \\ 0 & -2 \end{bmatrix} \right)^{-1} \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \]
\[ = \left[ 1 \ 1 \right] \left( \begin{bmatrix} s + 3 & -1 \\ 0 & s + 2 \end{bmatrix} \right)^{-1} \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \]
\[ = \left[ 1 \ 1 \right] \frac{1}{(s + 3)(s + 2)} \begin{bmatrix} s + 2 & 1 \\ 0 & s + 3 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \]
\[ = \frac{3s + 10}{(s + 3)(s + 2)}. \]

This coincides with the answer from method 1.

(b) The poles of the system are at \(-3\) and \(-2\). Since the poles of the system lie in the left half plane, the given system is stable.

2) Solution:

We first notice that
\[ L[y(t)] = Y(s) = H(s)L[u(t)] = H(s) \frac{1}{s}, \]
\[ \Rightarrow H(s) = sY(s) \]

Hence
\[ H(s) = s \left[ \frac{5}{s} - \frac{2}{s + 3} - \frac{3}{s + 6} \right] = 6 \frac{4s + 15}{s(s + 3)(s + 6)} \]

Thus the system has three poles at \(s1=0\), \(s2=-3\), and \(s3=-6\), one zero located at \(s=-15/4\).
3) Solution:

(a) The transfer function is given by:

\[ TF = \frac{Y(s)}{R(s)} = \frac{k}{(s + 4)(s + b) + kK} = \frac{k}{s^2 + (4 + b)s + 4b + kK} \]  \hspace{1cm} (1)

Since \( R(s) = 1/s \) (unit step input), the steady state value of the output is:

\[ SS = \lim_{s \to 0} (TF) = \frac{k}{4b + kK} \]  \hspace{1cm} (2)

From the figure for the step response, \( SS = 1 \), so, equation (2) becomes:

\[ \frac{k}{4b + kK} = 1 \Rightarrow k = 4b + kK \]  \hspace{1cm} (3)

Comparing equation (1) with the standard expression for the transfer function of a second order system gives

\[ \omega_n^2 = k = 4b + kK \quad \text{and} \quad 2\xi\omega_n = 4 + b \]  \hspace{1cm} (4)

From the figure for the step response, the peak time is given by:

\[ t_p = 0.49 \, s \]
The maximum overshoot is:

\[ Mp = \frac{1.23 - 1}{1} = 0.23 \]

The times for amplitude values of 10% and 90% \(SS\) can be obtained from the figure are approximately 0.08s and 0.355 s, respectively, so the rise time is:

\[ tr = 0.282 - 0.0687 = 0.2133 \text{ s} \]

With the maximum overshoot we get the damping factor:

\[ \xi = -\frac{\ln M_p}{\sqrt{\pi^2 + (\ln M_p)^2}} = -\frac{\ln 0.23}{\sqrt{\pi^2 + (\ln 0.23)^2}} = 0.4237 \]

The damped frequency is related to the peak time, the damping factor, and the natural frequency by:

\[ \omega_d = \frac{\pi}{t_p} \sqrt{1 - \xi^2} \rightarrow \omega_n = \frac{\pi}{t_p \sqrt{1 - \xi^2}} \rightarrow \omega_n = \frac{\pi}{0.49 \sqrt{1 - 0.4237^2}} = 7.074 \text{ rad/s} \]

With equations (4) we obtain \(k\) and \(b\):

\[ k = \omega_n^2 = (7.074)^2 = 50 \text{ rad}^2/\text{s}^2 \]

\[ b = 2\xi \omega_n - 4 = 2 \]

\[ K = 1 - \frac{4b}{k} = 1 - \frac{8}{50} = 0.84 \]

(b) On the figure for the step response, point (A) gives the position where the amplitude reaches 99% of \(SS\), therefore the settling time for 1% criterion is:

\[ t_s = 1.23 \text{ s} \]

4) Solution:

We first note that if \( b > 0 \), the unit step response is always positive, thus no undershoot occurs. To have undershoot in the unit step response it is sufficient that a linear system have a real zero in the open RHP. To investigate that possibility we add the two fractions in \(H(s)\) to yield
\[
H(s) = \frac{2s + 4 + bs + b}{(s + 1)(s + 2)} = (4 + b)\frac{2 + b}{4 + b} + \frac{1}{(s + 1)(s + 2)}.
\]

Then we have a Non-Minimum Phase (NMP) zero if and only if \(-4 < b < -2\).

5) Solution:

By block diagram algebra:

\[
Q = R - YH_3 - YH_4 = R - Y(H_3 + H_4)
\]

\[
Y = G_1 NG_6
\]

\[
\Rightarrow \frac{Y}{R} = \frac{G_1 NG_6}{1 + (H_3 + H_4)G_1 NG_6}
\]

And \(N\) is determined by
Move $G_4$ and $G_5$

Combine symmetric loops as in the first step:

Finally, we have

\[
N = \frac{O}{I} = \frac{G_4G_2 + G_5G_3}{1 + H_2(G_4 + G_5)}
\]

\[
\frac{Y(s)}{R(s)} = \frac{G_1(G_4G_2 + G_5G_3)G_6}{1 + H_2(G_4 + G_5) + (H_3 + H_4)G_1(G_4G_2 + G_5G_3)G_6}
\]
By Mason's Rule:
Signal flow graph is shown in below

Flow graph for Fig. 3.56

<table>
<thead>
<tr>
<th>Forward Path</th>
<th>Gain</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 2 3 4 5</td>
<td>( G_1 G_2 G_4 G_6 )</td>
</tr>
<tr>
<td>1 2 6 4 5</td>
<td>( G_1 G_2 G_5 G_6 )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Loop Path</th>
<th>Gain</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 2 3 4 5 1</td>
<td>(-G_1 G_2 G_4 G_6 H_3)</td>
</tr>
<tr>
<td>1 2 3 4 5 1</td>
<td>(-G_1 G_2 G_4 G_6 H_4)</td>
</tr>
<tr>
<td>1 2 6 3 5 1</td>
<td>(-G_1 G_2 G_5 G_6 H_2)</td>
</tr>
<tr>
<td>1 2 6 4 5 1</td>
<td>(-G_1 G_2 G_5 G_6 H_4)</td>
</tr>
<tr>
<td>3 4 3</td>
<td>(-G_4 H_2)</td>
</tr>
<tr>
<td>3 4 3</td>
<td>(-G_5 H_2)</td>
</tr>
</tbody>
</table>

and the determinants are

\[
\Delta = 1 + [(H_3 + H_4)G_1(G_2G_4 + G_3G_5)G_6 + H_2(G_4 + G_5)]
\]

\[
\Delta_1 = 1 - (0)
\]

\[
\Delta_2 = 1 - (0)
\]

\[
\Delta_3 = 1 - (0)
\]

\[
\Delta_4 = 1 - (0)
\]

Applying the rule, the transfer function is

\[
\frac{Y(s)}{R(s)} = \frac{1}{\Delta} \sum G_i \Delta_i
\]

\[
= \frac{G_1(G_4G_2 + G_5G_3)G_6}{1 + H_2(G_4 + G_5) + (H_3 + H_4)G_1(G_4G_2 + G_5G_3)G_6}
\]
6) Solution:

The system d.c. gain is given by $H(0)$ where $H(s)$ is the system transfer function, which is equal the Laplace transformation of $h(t)$, i.e. the Laplace transform of the system response to a unit impulse. Therefore

$$
H(0) = \int_{-\infty}^{\infty} h(t)dt = \int_{0}^{\infty} 2e^{-3t}dt = \frac{2}{3}
$$

7) Solution:

The system has relative degree 3 with 3 poles - hence it has no finite zeros.

It has 3 poles, hence it takes the form

$$G(s) = \frac{K}{A(s)(s + 2)(s + 4)}$$

Since the impulse response resembles a step response with steady state value we conclude the system must contain a pole at zero. Therefore the transfer function is of the form

$$G(s) = \frac{K}{s(s + 2)(s + 4)}$$

Then we use the final value theorem to determine $K$.

$$\lim_{s \to 0} G(s) = \lim_{s \to 0} \frac{sK}{s(s + 2)(s + 4)} = \frac{K}{8}$$

Given the steady state value of 0.25, i.e. $\lim_{s \to 0} sG(s) = 0.25$ we have

$$K = 2$$

Therefore

$$G(s) = \frac{2}{s(s + 2)(s + 4)}$$