Homework Set # 7 – Due July 23, 2004 @ 5:00 PM

1) A dynamical system has input $u$, output $y$ and transfer function

$$G(s) = \frac{Y(s)}{U(s)} = \frac{2s + 10}{5s^3 + 3s^2 + s + 4}.$$ 

For the control canonical form of the state equations

$$\dot{X} = AX + Bu, \quad Y = CX,$$

Determine the matrix $A$, and the vectors $B$ and $C$.

2) Consider the state space models with the following $A$, $B$ matrices

$$(1) \quad A = \begin{bmatrix} -3 & 1 & 1 & 2 \\ -2 & 3 & 1 & 2 \\ 2 & 1 & 4 & 1 \\ -4 & -3 & -1 & 0 \end{bmatrix}; \quad B = \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix},$$ 

$$(2) \quad A = \begin{bmatrix} -1 & 1 \\ 2 & 0 \end{bmatrix}; \quad B = \begin{bmatrix} 1 \\ -1 \end{bmatrix}. $$

Compute the relative controllability index of each mode in these two systems. Are these two systems stabilizable? Justify your answers.

3) Consider the state space models with the following $A$, $B$, $C$ matrices

$$(1) \quad A = \begin{bmatrix} -1 & 2 \\ 1 & 0 \end{bmatrix}; \quad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}; \quad C = \begin{bmatrix} 1 & -1 \end{bmatrix};$$ 

$$(2) \quad A = \begin{bmatrix} -3 & -2 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -5 \end{bmatrix}; \quad B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}; \quad C = \begin{bmatrix} 1 & -1 & -1 \end{bmatrix}. $$

Compute the relative observability index of each mode in these two systems. Are these two systems detectable? Justify your answers.

4) A linear continuous-time system is described in state space form using the following matrices

$$A = \begin{bmatrix} -2 & \epsilon \\ 4 & -5 \end{bmatrix}; \quad B = \begin{bmatrix} 0 \\ 3 \end{bmatrix}; \quad C = \begin{bmatrix} 1 & -1 \end{bmatrix}; \quad D = 0$$

(a) Show that the system is controllable if and only if $\epsilon \neq 0$. 
(b) Compute the transfer function, \( G(s) \), from \( U(s) \) to \( Y(s) \), and show that there is a pole-zero cancellation if \( \varepsilon = 0 \) (sufficiency). Is this a necessary condition? Justify your answer.

5) A continuous-time system has a state space model given by the following matrices
\[
A = \begin{bmatrix}
-2 & 3 & 0 \\
1 & 0 & 0 \\
0 & 0 & 4
\end{bmatrix} ; \quad B = \begin{bmatrix}
1 \\
1 \\
1
\end{bmatrix} \\
C = \begin{bmatrix}
1 & -1 & 1
\end{bmatrix} ; \quad D = 0
\]

(a) Determine whether or not the system is stable.

(b) Investigate the system “-ability” properties (i.e., controllability, stabilizability, observability, and detectability).

6) For a system with the following state equations and zero initial condition,
\[
\dot{X} = \begin{bmatrix}
-5 & 1 \\
-3 & -1
\end{bmatrix} X + \begin{bmatrix}
0 \\
1
\end{bmatrix} u \\
Y = \begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix} X
\]

find the steady-state value of \( y(t) \) for a step input \( u(t) \).