Homework # 2– Due June 18, 2004 @ 5:00 PM

1) Write the equations of motion for the double-pendulum system shown in Fig.1. Assume the displacement angles of the pendulums are small enough to ensure that the spring is always horizontal. The pendulum rods are assumed massless with length L, and the springs are attached $\frac{3}{4}$ of the way down.

![Fig. 1 Double Pendulum](image)

2) Obtain the state equations for the electric circuit shown in Fig.2, the output is the voltage of C1. The input or control variable is a current source $i(t)$.

![Fig. 2 Electric Circuit of problem2](image)

3) A coupled electromechanical system is shown in Fig.3. Electric potential $v$ creates a mechanical strain $F_m$. The vibration of mechanical system creates current $i$ in the electrical system. The electromechanical coupling is given by the relations:

\[ F_m = TV \]
\[ i = TV_m \]
where $F_m$ is the strain force on the Mass $M$ and $V_m$ is the velocity of $M$.

(a) Identify the inputs and state variables of the given electromechanical system.
(b) Write the dynamic equations for the electrical system.
(c) Write the dynamic equations for the mechanical system.
(d) Write the state-space representation of the complete system.

4) Derive the equations of motion for the inverter pendulum problem shown in Fig. 4.

The length of the pendulum varies with time while one attempts to control the system, whereas the pendulum mass remains constant. (Do not assume small angle approximation.)

Hint: The length of the pendulum $l$ should be a variable. There should be $\dot{l}$ and $\ddot{l}$ terms in the final equations of motion.
Description of the variables used

\( \dot{x} \) – Constant input force
\( M \) – Mass of the slider
\( m \) – Mass of the pendulum
\( l \) – Pendulum length
\( b \) – Coefficient of friction

5) The governing differential equations are given by

\[
\begin{align*}
\dot{x}(t) &= y(t)\left[1 - x^2(t)\right] \\
\dot{y}(t) &= x(t)\left[1 + y^3(t)\right]
\end{align*}
\]

(a) Determine the equilibrium points of this system. Is the equilibrium point unique?
(b) Linearize the given non-linear equation about the equilibrium point \( x = -1 \) and \( y = -1 \).


7) The governing differential equations of motion for a system are given as follows:
Fig. 7. Problem 7

\[
(I + ml^2)\dot{\alpha}(t) + mgl \sin \alpha(t) + ml\ddot{x}(t) \cos \alpha(t) = 0,
\]

\[
(m + M)\ddot{x}(t) + b\dot{x}(t) + kx(t) + ml\dddot{x}(t) \cos \alpha(t) - ml[\dot{\alpha}(t)]^2 \sin \alpha(t) = F(t)
\]

(a) Determine the operating point of the system described above with \( F_0 = 0 \);
(b) Linearize the differential equations governing its dynamics about this operating point.