Homework # 1– Due June 11, 2004 @ 5:00 PM

1) Find the Fourier Transform of the half-wave:

\[ \omega = \frac{2\pi}{T} \]

\[-T/4 \quad T/4\]

2) Evaluate the Fourier transform of the following continuous signal

\[ f(x) = U(x)Ae^{-ax}, \quad \text{for} \quad a > 0 \]

The function \( U(x) \) is defined as follows:

\[ U(x) = \begin{cases} 1, & x \geq 0 \\ 0, & x < 0 \end{cases} \]

3) Find the inverse transform of the following Discrete-time Fourier Spectrum:

\[ X(\omega) = \begin{cases} 1, & \text{for} |\omega| \leq \frac{\pi}{3} \\ 0, & \text{otherwise} \end{cases} \]

4) Obtain the Laplace transform of the following function.

\[ f(t) = 1 + t \sin(t); \quad t \geq 0 \]

5) Obtain the Laplace transform of the following function.

\[ f(t) = te^{-at} \]

6) Consider the following continuous signal:

\[ x_a(t) = 3\cos(500\pi t) + 6\cos(1000\pi t) + 8\cos(2000\pi t) \]
a. Determine the minimum frequency required for use in sampling the above signal in order to avoid aliasing.
b. If the sampling frequency is chosen as \( F_s = 1000 \) Hz, then what is the maximum frequency that can be recovered from the resulting discrete-time signal?

7) An analog signal \( x_a(t) = \sin(20\pi t) + 3\sin(140\pi t) + \cos(160\pi t) \) is sampled at 200 Hz.

a. Compute the discrete Fourier transform (Hint: Use the MATLAB function, ‘fft’) and plot the magnitude of the signal with respect to the frequency when the signal is sampled for
   (1) 1 second,
   (2) 10 seconds,
   (3) 100 seconds,
   Comment on the differences you observe in your plots.

b. For case (3) of part (a), downsample the signal (Hint: Use the MATLAB function ‘dyaddown’) to a sampling frequency of 100 Hz without passing the signal through a low pass filter. Plot the magnitude of the resulting signal with respect to the frequency. What do you observe? Is there aliasing? If yes, explain in detail how to obtain the aliased frequencies.

c. For the case (3) of part (a), pass the signal through a low pass filter and then downsample the signal (Hint: Use the MATLAB function ‘resample’) to a sampling frequency of 100 Hz. Plot the magnitude of the resulting signal with respect to the frequency. What do you conclude from the plot? Explain in detail.