Example 1:

Design a lead compensation for the system given by the transfer function

\[ G(s) = \frac{1}{s(s + 1)} \], that will provide a closed-loop damping \( \zeta > 0.5 \) and natural frequency \( \omega_n > 7 \text{ rad/sec} \).

**Sol:** The general transfer function of a lead compensator is given as

\[ D(s) = K \frac{(s + z)}{(s + p)}, \ p > z. \]

Let us design for the limiting condition of the damping ratio and natural frequency. Therefore let us choose \( \zeta = 0.5 \) and \( \omega_n = 7 \text{ rad/sec} \). Hence the closed loop poles of the system are given by

\[
-\zeta \omega_n \pm j\omega_n \sqrt{1 - \zeta^2},
\]

\[
= -3.5 \pm j6.062
\]

Let us choose \( z = 2 \). The angle subtended by all the poles and zeros of the feed forward transfer function, with the closed loop pole at \(-3.5 + 6.062j\) is

\[ -\theta_p = 112.41 \text{ radians} - 120 + 103.898 \]. This angle must be equal to \(-180\) degrees. Hence the angle subtended by the compensator pole with the closed loop pole is 51.488 degrees.
Therefore,
\[ \tan(51.488) = \frac{6.062}{x}, \]
\[ \Rightarrow x = 4.824. \]

\[ p = x + 3.5 = 4.824 + 3.5 = 8.324 \approx 9. \]

To calculate the gain \( K \), we have
\[ 1 + D(s)G(s) = 0, \]
\[ \Rightarrow |D(s)G(s)| = 1 \]
\[ \Rightarrow K \left( \frac{s + 2}{(s + 9)(s + 1)} \right)_{s = 3.5 + 6.062j} = 1, \]
\[ \Rightarrow K \approx 60. \]

Hence the lead compensator is given by
\[ D(s) = 60 \frac{(s + 2)}{(s + 9)}. \]

Plotting the root locus of the feed forward transfer function given by \( D(s)G(s) \), the specification of the location of the closed-loop pole can be verified.

**Example 4:**

Consider the system whose feed forward transfer function is given by
\[ G(s) = \frac{K}{s(s + 2)}. \] Design a lag compensator so that the dominant poles of the closed loop system are located at \( s = -1 \pm j \) and the steady state error to a unit ramp input is less than 0.2.

**Sol:**

The general transfer function for lag compensation is given by
\[ D(s) = \frac{(s + z)}{(s + p)}, \quad p < z \]

The forward transfer function is given as
For the specification that the steady state error of the system must not exceed 0.2, we have

\[ E(s) = \frac{s(s+2)(s+p)}{s(s+2)(s+p)+K(s+z)} R(s), \]

\[ \Rightarrow e_{ss} = \lim_{s \to 0} sE(s) = \lim_{s \to 0} s \left[ \frac{s(s+2)(s+p)}{s(s+2)(s+p)+K(s+z)} R(s) \right]. \]

For a ramp input, we have

\[ e_{ss} = \lim_{s \to 0} \left[ \frac{s(s+2)(s+p)}{s(s+2)(s+p)+K(s+z)} \right] \frac{1}{s^2} = \frac{2p}{Kz} < 0.2. \]

Let \( \frac{2p}{Kz} = 0.2 \)

Let us choose \( p = 0.01 \), therefore we have \( Kz = 0.1 \)

We know that, the closed loop poles lie in the root locus and hence

\[ 1 + D(s)G(s) = 0, \]

\[ \Rightarrow K = -\left. \frac{1}{D(s)G(s)} \right|_{s=-1+j}, \]

\[ \Rightarrow K = -\left. \frac{s(s+2)(s+0.01)}{(s+z)} \right|_{s=-1+j}. \]

Solving for \( K \) and \( z \), we get

\( K = 1.88 \) and since \( Kz = 0.1 \), we get \( z = 0.0532 \).

Therefore the lag compensator is given by

\[ D(s) = \frac{(s+0.0532)}{(s+0.01)}. \]

Plotting the root locus of the feed forward transfer function, given by \( D(s)*G(s) \), the specification of the location of the closed loop pole can be verified.
Recommended Reading


Recommended Assignment